Observation of modulationally unstable multi-wave mixing

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We demonstrate experimentally that multiple four-wave mixing (FWM) pumped by a dual-frequency input in a single-mode fiber is modulationally unstable. This collective type of instability leads, in the anomalous dispersion regime, to sideband growth around all orders of FWM. This is in contrast with the normal dispersion regime where our measurements show that FWM exhibits no instability. Our conclusions are based on the first systematic mapping of the phenomenon as a function of the dual-pump input frequency separation.

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Modulational instability (MI) is a well known process [1], where a modulation at frequency Ω_m grows exponentially at the expense of a pump wave (carrier eigenmode) at frequency ω_0. When such a modulation grows out of noise (spontaneous MI), its peak frequency depends on power as determined by the nonlinear phase-matching condition, which for a scalar field requires anomalous group-velocity dispersion (GVD). While MI is still a subject of active investigation in this regime [2,3], it has also been considerably generalized to include two-component pump eigenmodes (incoherently coupled pumps) such as in the polarization case studied experimentally under different conditions [4,5]. A further noteworthy generalization of MI entails the growth of sidebands on top of two pump components which exchange energy in a periodic fashion, as originally predicted in [6,7] and first proven experimentally in quadratic media [8]. Interestingly enough, in this case the MI process can be seen as a parametric resonance in a system where the longitudinal periodicity is intrinsically built-in owing to the pump dynamics [9]. An interaction which also falls under this class of problems is multiple four-wave mixing (mFWM) pumped by a dual-frequency input beam [10]. In fact, when two co-polarized pump frequencies $\omega_\pm = \omega_0 \pm \Omega_p$ (pump detuning $2\Omega_p$) are launched along an optical fiber, they produce mFWM, i.e., generation of the odd harmonics of the input modulation, namely $\omega_0 \pm n\Omega_p$ with $n = 3, 5, \ldots$ [11–14] (note that, unlike the standard MI, in this case no pump is present at $\omega_0 = (\omega_+ + \omega_-)/2$, which is used only as the reference central frequency of the generated mFWM spectrum). The mFWM process is characteristic of any Kerr media and can be observed also in other settings, e.g., semiconductor photonic crystal guides [15], or in the spatial domain [16]. mFWM is exploited for important applications, which range from signal regeneration [17] to parametric amplification [18] and pulse-train generation [13,19,20], and is also being investigated with reference to fundamental phenomena [21].

It has been recently shown that the recurrent evolution of the pump and four-wave mixing (FWM) sidebands allows for MI to manifest, in the anomalous GVD regime, as the exponential growth of an extra-modulation at frequency Ω_m of the primary modulation and its mFWM harmonics, owing to the generation of photon pairs at frequency $\omega_0 \pm n\Omega_p \pm \Omega_m$ [10]. This prediction relies on the extension of the approach to stability to account for FWM via a Floquet type of analysis [10], thus considerably improving previous approaches based on incoherently coupled nonlinear Schrödinger (NLS) equations [22,23], which were argued to neglect FWM [24]. The underlying mechanism of this MI process is the fact that the two pump frequencies $\omega_\pm$ are unstable, provided the GVD is anomalous, and transfer their modulation (arising from the growth of frequency Ω_m from noise) over all the mFWM products. However, at variance with conventional scalar MI, in this case the net amplification of the extra-modulation occurs over several periods of conversion and backconversion of mFWM. Moreover, mFWM sidebands do not acquire their modulation directly because otherwise they would be modulated at a different frequency owing to their much lower power. Because of these features we denoted this phenomenon as collective MI. The purpose of this Letter is to report the first experimental observation of this process in a standard telecom fiber by means of a systematic mapping of the phenomenon as a function of the initial two pumps frequency detuning.

The experimental setup developed in order to characterize the collective nature of the MI building on top of mFWM process in fibers is illustrated in Fig. 1. It consists of two external cavity lasers (ECLs) centered around $\lambda_0 = 1550$ nm and coupled by means of a 50:50 coupler, with remotely adjustable relative frequency detuning. A phase modulator (PM), driven by a 100 MHz RF signal is then used to prevent any Brillouin back-scattering effect in the fiber under test. The dual-pumps are then amplified by means of an Erbium doped fiber amplifier (EDFA) to reach a total average power of 800 mW. A 5 nm optical filter is also inserted to limit the impact of amplified spontaneous emission from the EDFA before injection into a 10 km long standard single mode

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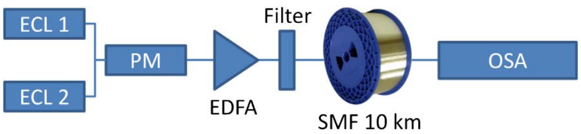

Fig. 1. (Color online) Experimental setup. External cavity laser (ECL), phase modulator (PM), and optical spectrum analyzer (OSA).

fiber (SMF-28). The SMF has physical parameters as follows: the dispersion is $D = 17$ ps/nm km with a slope $S = dD/d\lambda = 0.05$ ps/nm^2 km, the nonlinear Kerr coefficient is $\gamma = 2\pi n_{2I}/\lambda_0 A_{\mathrm{eff}} = 1.3$ W^{-1} km^{-1}, and the linear loss is $\alpha = 0.2$ dB/km. At the output of the fiber, the signal is then characterized in the spectral domain by means of an optical spectrum analyzer (OSA). Thanks to a home-made remote control algorithm developed in a Labview environment, a spectral mapping of the phenomenon is achieved by a synchronization procedure which allows to monitor the resulting output spectrum as a function of the frequency detuning between the two pumps in a range limited by the amplifier bandwidth and for a constant average power.

The results of our experiments, obtained for a constant total average power $P = 800$ mW, are summarized in Fig. 2. Figure 2(a) shows a typical output spectrum obtained for a balanced input (400 mW on each pump) detuned by $\Delta f_p = \Omega_p/\pi = 100$ GHz (in order to compare with the normalized units used in [10], this corresponds to a pump detuning $2\Omega = 2.8$). As shown in Fig. 2(a), MI sidebands corresponding to an extra-modulation at frequency $f_m = 34$ GHz grows spontaneously from noise over the mFWM. In Fig. 2(a), the MI sidebands owing to such extra-modulation are clearly seen around $n = 1, 3, 5$ FWM sidebands. The Floquet stability analysis [10] of the FWM predicts that the maximally unstable modulation frequency turns out to coincide with the peak gain frequency of the standard scalar MI associated with a single pump. Starting from the fiber parameters, namely a GVD $k'' \simeq -22$ ps^2/km and a nonlinear length $Z_{\mathrm{nl}} = (\gamma P/2)^{-1} \simeq 1.9$ Km associated with the pump power $P/2 = 400$ mW, we estimate such frequency to be $f_{\mathrm{MI}} = \sqrt{2/(|k''|Z_{\mathrm{nl}})}/(2\pi) = 34.86$ GHz, in good agreement with the value observed from the spectra. It is important, however, to emphasize that the phenomenon possesses an *intrinsic collective nature*, with the same modulation frequency f_m growing on top of the pumps and higher FWM orders, as well. Assuming, viceversa, that MI could develop around e.g., the first-order FWM sideband ($n = 3$) independently from the modulation acquired by the pumps, it should have been observed that such sidebands develop a modulation one order of magnitude slower according to their power level, which is −20 dB below the pump power, [see Fig. 2(a)].

In order to investigate the dependence of collective MI on the pump detuning, we have also recorded spectra [such as the one in Fig. 2(a)] for different pump detunings. In particular our setup allows one to tune the wavelength detuning between the lasers in steps of $\Delta\lambda = 0.01$ nm over a whole range, which is equivalent to frequency detunings ranging from 250 down to −10 GHz (so to include as a reference $\Delta f_p = 0$, where FWM is expected to vanish), while keeping fixed the injected power at 800 mW. The result is illustrated in the (color) level map in Fig. 2(b). First, mFWM are clearly observed to correspond to the diagonal brighter narrow lines, which grows in number as the detuning Ω_p is decreased (mFWM becomes more and more efficient as its figure of merit $\gamma P/(|k''|\Omega_p^2)$ grows larger [13]), until a strong spectral broadening due to mFWM explosion is observed just near the dark point, which corresponds to $\Delta f_p = 0$, where mFWM is found indeed to vanish. From the map we clearly see that the collective MI frequency f_m remains locked to the value f_{MI} (see empty circles in the figure), basically not exhibiting any dependance on the pump detuning, as expected from the linear stability analysis. Sidebands due to collective MI remain clearly visible in Fig. 2(b) in the range 70–250 GHz. They disappear when they coalescence with the mFWM sidebands, which occurs around a pump detuning $\Omega_p \simeq 70$ GHz. At lower detunings, not only does the MI become resonant with the mFWM but also the latter looses its features of recurrence [13]. Under such conditions the Floquet approach looses its validity and the assessment of the linear stability problem in the presence of (highly efficient) mFWM still remains a challenging open problem that will require new approaches.

We have also studied the robustness of the phenomenon against the imbalance of the pumps. Figures 2(c)

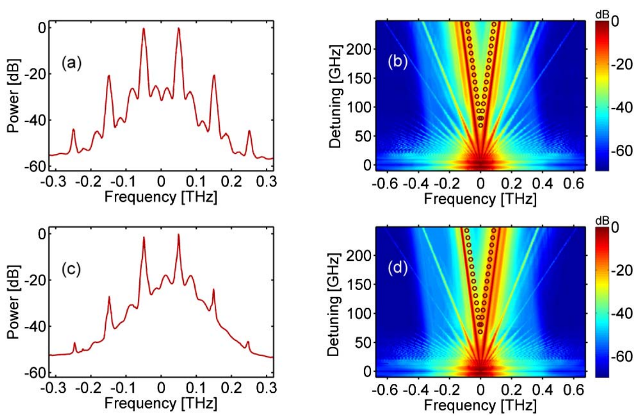

Fig. 2. (Color online) Experimental output spectra showing MI developing over mFWM for balanced (a) and imbalanced (c) pumps at $\Delta f_p = 100$ GHz and $P = 800$ mW, color maps of output spectrum as a function of detuning Δf_p ranging from −10 to 250 GHz, measured in the balanced (b), and imbalanced case (d). Open dots correspond to the most unstable frequency f_{MI} from theory.

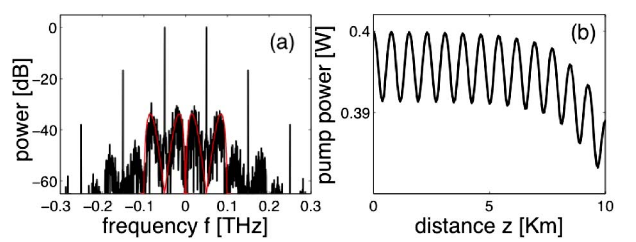

Fig. 3. (Color online) Output spectrum calculated from Eq. (1) with same parameters as in Fig. 2(a) (a). The MI gain is also reported around the pumps for comparison (thin red line, vertical a.u.) (b). Corresponding evolution along the fiber of the power of one of the pumps.

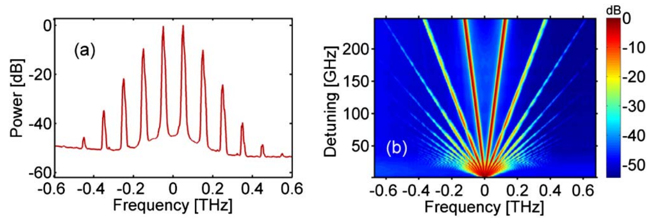

Fig. 4. (Color online) Same as in Figs. 2(a) and 2(b) for balanced input in the normal GVD regime.

and 2(d) display a typical output spectrum and the relative map against the pump detuning obtained when the pumps are imbalanced by about 10% (input power fractions $\eta = P_+/P = 0.56$ and $1 - \eta = 0.44$, respectively). As shown the collective MI is still visible, though the sidebands due to the extra-modulation at frequency f_m are more clearly pronounced around the stronger pump. In our experiment we have found that the MI spectrum is very sensitive to the pump imbalance and tends to disappear for stronger asymmetries.

The collective MI process can be described by means of a single NLS equation, whose nonlinear term contains all the beating products that give rise to mFWM orders. In order to assess whether a quantitative agreement exists, we have performed simulations of the NLS equation

$$i\frac{\partial E}{\partial Z} - \frac{k''}{2}\frac{\partial^2 E}{\partial T^2} + \gamma|E|^2 E = -i\frac{\alpha}{2}E, \tag{1}$$

using the parameters of the fiber and the input $E_0(T) = \sqrt{P}[\sqrt{\eta}\exp(i\pi\Delta f_p T) + \sqrt{1-\eta}\exp(-i\pi\Delta f_p T)]$ in the presence of white noise. A typical spectrum obtained with the same parameters as shown in Fig. 2(a) is reported in Fig. 3(a) for the balanced case $\eta = 0.5$ (similar results are obtained in the unbalanced case). The comparison between Figs. 2(a) and 3(a) allows us to conclude that a satisfactory quantitative agreement exists. As shown in Fig. 3(b) MI develops on top of a periodic evolution, from which the system adiabatically decays as soon as the MI leads to a substantial amplification of the extra-modulation, a feature which, however, we are not able to measure.

Finally we have also investigated experimentally the same phenomenon in the normal GVD regime. To access this regime we have replaced the SMF with a 6 Km long non-zero dispersion-shifted fiber (NZDSF) with dispersion $D = -2.5$ ps/nm km (slope $S = 0.07$ ps/nm^2 km), nonlinear Kerr coefficient $\gamma = 1.7$ W^{-1} km^{-1}, and linear loss coefficient is $\alpha = 0.2$ dB/km. We report a typical spectrum obtained for $\Delta f_p = 100$ GHz and $P = 800$ mW and the relative map against the pump detuning in Figs. 4(a) and 4(b), respectively. By comparing Figs. 4(a) and 4(b) with Figs. 2(a) and 2(b), it is clear that, in the normal GVD regime, the dynamics is fully dominated by mFWM even at large detunings and we observe no sidebands arising from collective MI, as anticipated on the basis of the linear stability analysis [10].

In summary, our experiments show that mFWM mixing exhibits, in the anomalous GVD regime, the onset of collective MI process, whose signature is the appearance of an extra-modulation at fixed frequency around all orders of the primary mixing process.

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